Advanced control with a Cooper-pair box: Stimulated Raman adiabatic passage and Fock-state generation in a nanomechanical resonator

Jens Siewert,[1,2] Tobias Brandes,[3,*] and G. Falci[1]
[1]*Dipartimento di Metodologie Fisiche e Chimiche per l'Ingegneria and MATIS-INFM, Consiglio Nazionale delle Ricerche, Universita di Catania, I-95125 Catania, Italy*
[2]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[3]*The University of Manchester, School of Physics and Astronomy, P.O. Box 88, Manchester M60 1QD, United Kingdom*
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The rapid experimental progress in the field of superconducting nanocircuits gives rise to an increasing quest for advanced quantum-control techniques for these macroscopically coherent systems. Here we demonstrate theoretically that stimulated Raman adiabatic passage should be possible with the quantronium setup of a Cooper-pair box. The scheme appears to be robust against decoherence and should be realizable even with the existing technology. As an application we present a method to generate single-phonon states of a nanomechanical resonator by vacuum-stimulated adiabatic passage with the superconducting nanocircuit coupled to the resonator.

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I. INTRODUCTION

One of the most fascinating experimental breakthroughs of the recent past is the observation of coherent dynamics in superconducting nanocircuits. It includes circuits exhibiting the dynamics of single "artificial atoms,"[1–3] two coupled artificial atoms,[4,5] and artificial atoms coupled to electromagnetic resonators.[6–8] This development opens new perspectives to study quantum phenomena in solid-state devices that traditionally have been part of nuclear magnetic resonance, quantum optics, and cavity quantum electrodynamics. There exist already a number of theoretical proposals (as well as experimental realizations) for, e.g., the detection of geometric phases,[9,10] the preparation of Schrödinger cat states in electrical and nanomechanical resonators,[11,12] cooling techniques,[13] an analog of electromagnetically induced transparency,[14] and adiabatic passage in superconducting nanocircuits.[15–17]

An important aspect in these studies is to understand how flexibility in the design of devices and protocols may improve robustness of coherence against noise in the solid state. Compared to quantum optics, entirely new regimes of stronger coupling between the system elements and of stronger influence of noise, which in the solid state is significant also at low frequencies,[18] have to be explored.

A particularly robust and versatile protocol in atomic physics is the adiabatic population transfer between two hyperfine ground states $|g\rangle$ and $|u\rangle$ of a three-level atom that are coupled via two laser fields to an excited state $|e\rangle$ such that they form a Λ configuration. The protocol is named stimulated Raman adiabatic passage (STIRAP) (Ref. 19) and is closely linked with both Abelian and non-Abelian geometric operations[20] that may be applied for holonomic quantum computation.[21]

The demonstration of STIRAP in a nanodevice has its own interest since the macroscopic nature of the system and the peculiar effects of low-frequency noise in the solid-state impose obstacles requiring more careful studies than a mere translation from quantum optics. Moreover, STIRAP has interesting applications that may lead to solutions for challenges such as deterministic Fock state generation in solid-state resonators coupled to a nanocircuit. In quantum optics, single-photon generation has been achieved by replacing one of the external drives by the quantum field of a cavity.[22,23] Recently, single-photon generation triggered by Rabi oscillations in a circuit-QED (quantum electrodynamics) setup has been reported.[8]

In the following we will analyze STIRAP in a single Cooper-pair box (CPB) in the charge-phase regime[2] (the so-called quantronium qubit), operated as a three-level atom (Sec. II), pointing out the effect of decoherence (Sec. III). To include decoherence is *a priori* important because, as opposed to atomic systems, in solid-state analogs one of the two low-lying states involved in STIRAP is not a ground state and may decay into the other. In Sec. IV, we then show that the circuit is appropriate for the substitution of one of the classical driving fields by the quantum field of a coupled nanomechanical resonator without changing its functionality. Verification of the vacuum-assisted adiabatic passage with single-phonon state generation completes the analog of the atom-cavity system in Refs. 22 and 23.

We remark that in principle this program can be carried out for different regimes and setups of superconducting nanocircuits.[24] However, especially the effect of low-frequency noise may turn out detrimental in view of the sensitivity of STIRAP to parameters as the detunings.[19] Combined with energy relaxation, these effects may prevent realization of the protocol in solid-state implementations and therefore require careful assessment.

The CPB in the charge-phase regime offers a uniquely convenient design to demonstrate coherent population transfer in the solid state. The overall noise is minimized and the whole protocol is simplified since the two-port design allows us to operate this device analogously to an atom. It displays excellent properties regarding relaxation and dephasing rates: the deterioration of the qubit signal occurs on time scales equal or larger than the inverse decay constant of Ramsey fringes,[2,25] which in the charge-phase qubit may be made as large as $\tau_R \gtrsim 300$ ns by operating it at an optimal working point. This analysis applies also for STIRAP since we argue that *both high- and low-frequency noises* effective in this protocol involve mainly the two low-lying levels while fluctuations of the third level are less important. Finally, the charge-phase regime allows us to find a trade-off between the competing requirements of good protection against noise and efficient coupling to the driving field by tuning the device only slightly away from the optimal working point.

II. STIRAP IN THE QUANTRONIUM CIRCUIT

The Λ configuration needed for STIRAP is realized using two classical ac fields $A_g \cos \omega_g t$, $A_u \cos \omega_u t$, with single-photon detunings $\Delta_g = E_e - E_g - \omega_g$ and $\Delta_u = E_e - E_u - \omega_u$ (E_i, $i = g, u, e$ are the unperturbed CPB eigenenergies). If equal detunings $\Delta_g = \Delta_u = \Delta$ are chosen for the driving frequencies the two-photon resonance condition is fulfilled. In this case, retaining only corotating terms the three-level Hamiltonian in the rotating frame reads[19,26]

$$H_{\text{rot.f.}} = \Delta |e\rangle\langle e| + \frac{1}{2}(A_u |e\rangle\langle u| + A_g |e\rangle\langle g| + \text{H.c.}). \quad (1)$$

This Hamiltonian has an eigenvector called dark state,

$$|D\rangle = \frac{1}{\sqrt{|A_u|^2 + |A_g|^2}}(A_g |u\rangle - A_u |g\rangle). \quad (2)$$

In the counterintuitive scheme, the system is prepared in the state $|g\rangle$ with couplings $A_g = 0$ and $A_u \neq 0$, that is, the off-resonant ac drive does not produce any transition. For this configuration of couplings, the state $|g\rangle$ coincides with the dark state. Subsequently, the dark state is rotated in the two-dimensional subspace spanned by $|u\rangle$ and $|g\rangle$ by slowly varying in time the coupling strengths A_u and A_g. By switching A_u off while A_g is switched on, the population is transferred from state $|g\rangle$ to state $|u\rangle$. Adiabaticity requires $|\dot{A}_j / A_j| < \omega_j$ ($j = u, g$).

The Hamiltonian of the CPB in the laboratory frame is

$$H_{\text{CPB}} = \sum_n E_C [n - n_g(t)]^2 |n\rangle\langle n| - \frac{E_J}{2}(|n\rangle\langle n+1| + \text{H.c.}), \quad (3)$$

where $|n\rangle$ are charge eigenstates with n extra Cooper pairs in the island, E_C is the charging energy, and E_J is the Josephson coupling energy [see Fig. 1(a)]. For simplicity we assume $E_C = E_J$. We note that the analysis of the STIRAP protocol presented below holds for an entire range of ratios $E_J / E_C \gtrsim 1$, however, the deep transmon regime $E_J / E_C \gg 1$ requires special care due to the reduced anharmonicity of the corresponding CPB spectrum.[27] In the original setup,[2] the gate charge has a dc bias part n_{g0} and a single ac component $n_g^{\text{ac}} = A \cos \omega t$ with amplitude $|A| \ll 1/2$ allowing for controlled evolution in the (approximate) two-state system of the two eigenstates of lowest energy.

STIRAP can be carried out between the *three* lowest levels [see Fig. 1(b)]. By now it is well known[2] that the lowest

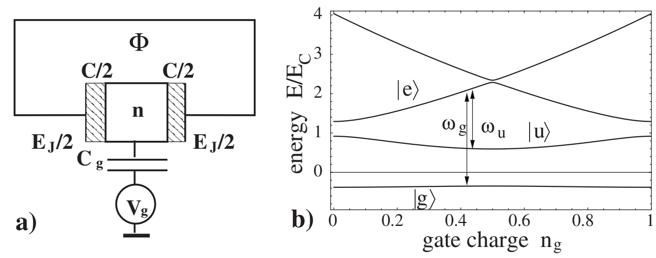

FIG. 1. (a) The Cooper pair box is a superconducting island of total capacitance C coupled to a superconducting lead via two Josephson junctions. The charging energy $E_C = 4e^2/(2C)$ sets the scale of the electrostatic energy, which is controlled by the gate charge $n_g = C_g V_g / (2e)$, where V_g is the gate voltage and $C_g \ll C$ is the gate capacitance. For the magnetic flux we choose $\Phi = 0$. (b) The lowest four energy levels of the CPB with $E_C = E_J$ as a function of the gate charge n_g. At the working point n_{g0} the three lowest levels can be used as a Λ scheme $|g\rangle$, $|u\rangle$, $|e\rangle$ with resonance frequencies $\hbar\omega_g = E_e - E_g$ and $\hbar\omega_u = E_e - E_u$.

decoherence rates are obtained if the system is biased at the symmetry point $n_{g0} = 1/2$. However, selection rules impede the operation of the three-level scheme.[16] This problem is circumvented by working slightly away from this point, e.g., at $n_{g0} = 0.45$. If two ac signals with slightly detuned frequencies ω_g and ω_u,

$$n_g(t) = n_{g0} + A_g(t) \cos \omega_g t + A_u(t) \cos \omega_u t, \quad (4)$$

are applied to the gate [see Fig. 1(b)], Hamiltonian (3) allows for nearly complete population transfer $|g\rangle \to |u\rangle$ as shown in Figs. 2(b)–2(d) where the numerical solution of the Schrödinger equation for Hamiltonian (3) is reported (solid lines). Notice that the microwave field couples *diagonally* to the charge states (as opposed to the dipole coupling for the three-level atom). Nevertheless, the effective Hamiltonian for the CPB eigenstates assumes the same form as in Eq. (1) since only those off-diagonal matrix elements in the eigenbasis of the driven Hamiltonian are important that couple two states resonantly.[28] The state $|e\rangle$ practically does not get populated during the STIRAP procedure [cf. Fig. 2(d)]. There are many parameters that may be used to optimize the efficiency such as duration, delay, relative height, and overall shape of the pulses, the detunings, etc.[19,29]

III. EFFECTS OF DECOHERENCE

The functionality of quantum-coherent nanodevices is sensitive to various (device-dependent) decoherence sources. In the charge-phase qubit, high-frequency noise (mainly responsible for unwanted transitions) coexists with low-frequency noise that mainly affects calibration of the device and determines power-law reduction in the signal amplitude.[18,25] Here we only assess the feasibility of the protocol; a detailed analysis of decoherence in the STIRAP protocol due to a solid-state environment will be presented elsewhere.

The key point is the observation that strong unwanted processes involving the level $|e\rangle$ have negligible effects on the dynamics, for *both* high- and low-frequency noises. De-

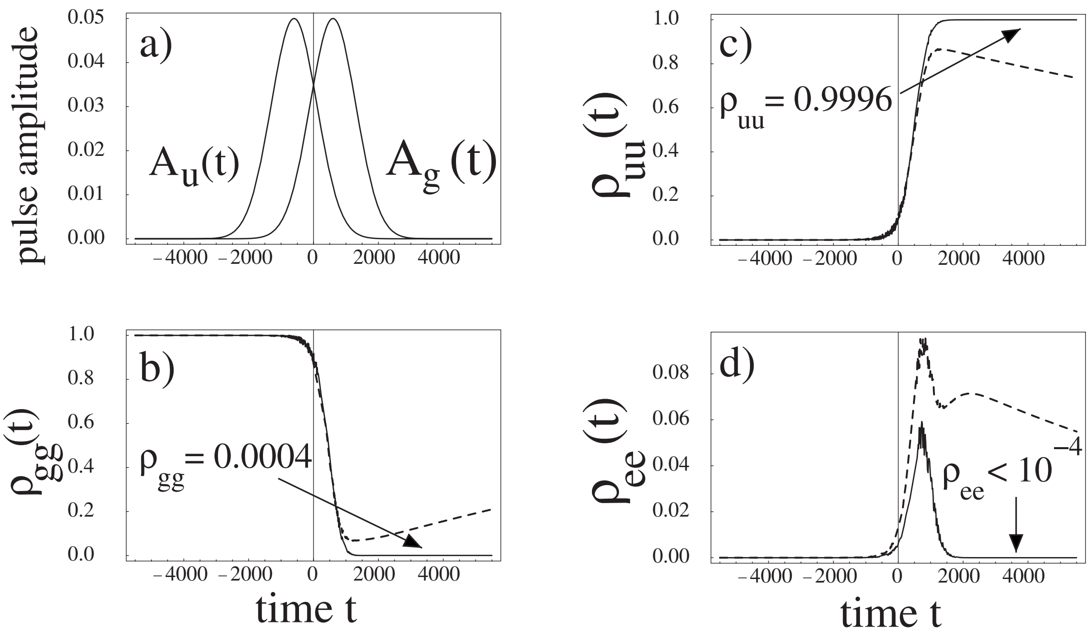

FIG. 2. Population transfer by STIRAP in the charge-phase qubit (n_{g0}=0.45). Varying the number of charge states between ten and four does not change the results. (a) Gaussian pulses $n_g(t)=n_{g0}+A_g(t)\cos\,\omega_g t+A_u(t)\cos\,\omega_u t$ with zero detuning are applied in the counterintuitive scheme. The maximum gate charge of the microwave fields are $\max[A_u(t),A_g(t)]=0.05 \ll 1/2$. For a charging energy of $E_C=50\ \mu$eV the time unit corresponds to about 1.3×10^{-11} s. [(b)–(d)] Time evolution of the populations ρ_{gg}, ρ_{uu}, ρ_{ee} for the isolated system (solid lines) for initial state $|g\rangle$. The arrows denote the final populations showing that selection rules, the presence of more than three levels and of diagonal and counter-rotating drives still allow for complete population transfer. We include quantum noise (dashed lines) using decay rates $\gamma_u=4.4\times 10^{-5}$ and a dephasing rate $\tilde{\gamma}_u=2.6\times 10^{-4}$ in E_C units, corresponding to a dephasing time of $\sim$50 ns. The only notable effect is the extra decay $|u\rangle\rightarrow|g\rangle$ at the end of the protocol.

coherence is mainly determined by processes involving $|g\rangle$ and $|u\rangle$, which have been well characterized and, as a matter of fact, allow for long decoherence times in charge-phase qubits.

The action of noise sources is analyzed by solving the quantum-optical master equation $\dot{\rho}=\frac{i}{\hbar}[\rho,H']-\Gamma\rho$, where ρ is the density matrix and H' is Hamiltonian (3) in the rotating frame.[30] At low temperature the dissipator $\Gamma\rho$ includes spontaneous decay and environment-assisted absorption between eigenstates in the presence of the laser coupling, and in the basis $\{|g\rangle,|u\rangle,|e\rangle\}$ reads

$$(\Gamma\rho)_{ij}=\frac{\gamma_i+\gamma_j}{2}\rho_{ij}-(1-\delta_{ij})\tilde{\gamma}\rho_{ij}-\delta_{ij}\sum_k \rho_{kk}\gamma_{k\rightarrow i}, \qquad (5)$$

where $\gamma_i=\Sigma_{k\neq i}\gamma_{i\rightarrow k}$. The dissipator is taken time independent (which overestimates decoherence) and includes all transitions as well as dephasing rates $\tilde{\gamma}$, accounting phenomenologically for low-frequency noise. For the second excited state we assume $\gamma_e=\gamma_{e\rightarrow u}+\gamma_{e\rightarrow g}=2\gamma_u$ where γ_u and $\tilde{\gamma}_u$ equal or larger than those observed in the experiments in Ref. 25 are used.

In quantum-optical systems most of the decay rates act on depopulated states and therefore are ineffective. This prevents in particular the strong decay from level $|e\rangle$ to enter the game. In solid-state devices there is the extra rate $\gamma_{u\rightarrow g}$ but it affects the populations only during the waiting time *after* completion of the pulse sequence [dashed lines in Figs. 2(b)–2(d)].

Solid-state noise may be strong down to low frequencies. It is mostly due to charged impurities which act as a stray polarization of the device,[18,25,31] determining fluctuations of the working point. The leading effect is due to impurities which are static during each run of the protocol but switch on a longer time scale.[18] This leads to statistically distributed level spacings, and averaging determines defocusing of the signal. This theory gives a quantitative description of dephasing of Ramsey fringes observed in the quantronium.[25]

For a continuously driven system, fluctuations δE_i translate in fluctuations of the single-photon detunings Δ_g and Δ_u in the rotating frame, which then determine the leading effect of low-frequency noise on STIRAP. This allows us to conclude that even larger δE_e hardly affect STIRAP since they maintain *equal single-photon detunings* of both microwave fields, thus preserving two-photon resonance. On the other hand, fluctuations of E_u-E_g are potentially detrimental since they translate in fluctuations of the difference $\Delta_g-\Delta_u$, leading to a random finite two-photon detuning.

Therefore the long dephasing time observed in the experiments[2,25] determines the time scale for STIRAP. Notice that a finite two-photon detuning due to fluctuations may give level crossing of the instantaneous eigenenergies. In this case population transfer is guaranteed by Zener tunneling, which still allows almost complete population transfer as long as the system is well protected (i.e., large two-photon detunings are rare). A larger noise level, typical of charge qubits and other implementations, determines instantaneous level repulsion eventually leading the system to the wrong final state.

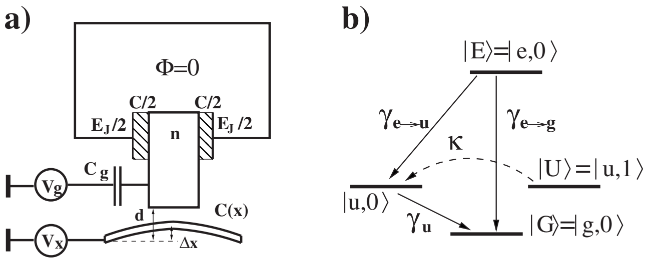

FIG. 3. (a) Coupled system of CPB and nanomechanical oscillator. (b) The four relevant states of the STIRAP scheme for Fock state generation in the presence of decoherence. Here κ denotes the oscillator decay rate.

IV. FOCK STATE GENERATION VIA STIRAP

Since STIRAP should be well within reach of present-day technology for superconducting nanocircuits one might hope to apply this technique for the preparation of peculiar quantum states. One such application is the generation of Fock states in a cavity coupled to a three-level atom.[22] To this end the CPB has to be coupled to a harmonic-oscillator degree of freedom by an interaction $H_{\rm int}=\lambda(a+a^\dagger)(n-n_g)$. This Hamiltonian can be implemented using electrical resonators,[32] transmission lines,[6,8] and nanomechanical resonators.[12,13]

In the following we explain how to generate Fock states in a nanomechanical oscillator (see also Ref. 33) of mass m coupled to the box[12,13] via the position-dependent capacitance $C(x)\simeq C_x+\Delta x[dC(x)/dx]$ [see Fig. 3(a)]. Here Δx denotes the oscillator displacement. Assuming $\Delta x \ll d$, where d is the distance of the resonator from the island, and taking into account only a single mode of the mechanical oscillator, the coupled CPB-resonator system is described by the Hamiltonian,[12,13]

$$H = H_{\rm CPB} + H_{\rm res} + H_{\rm int}, \quad H_{\rm res} = \hbar\omega_{\rm res} a^\dagger a,$$

$$H_{\rm int} = \frac{2E_C}{d} n_x \sqrt{\frac{\hbar}{2m\omega_{\rm res}}}(a+a^\dagger)(n-n_g-n_x)$$
$$\equiv \lambda(t)(a+a^\dagger)[n-n_g(t)-n_x(t)], \qquad (6)$$

where $a^\dagger$ and a are creation and annihilation operators for the nanomechanical oscillator, and the gate charge in $H_{\rm CPB}$ [Eq. (3)] has to be replaced with $n_g(t)+n_x(t) = n_{g0}(t)+A_g(t)\cos\,\omega_g t + C_x V_x(t)/(2e)$. That is, now there is only a single external gate pulse $A_g(t)$ and the coupling with the oscillator induces an additional gate charge $n_x(t) \equiv C_x V_x(t)/(2e)$, which can be tuned via the voltage V_x. The tunability of the CPB-oscillator coupling is the essential hardware feature to render this protocol feasible.

The basis for the composite system is $\{|j,N\rangle \equiv |j\rangle \otimes |N\rangle\}$ with the (uncoupled) CPB eigenstates $|j\rangle$ and the resonator Fock states $|N\rangle$. The states of the Λ configuration are $|G\rangle=|g,0\rangle$, $|U\rangle=|u,1\rangle$, and $|E\rangle=|e,0\rangle$.

We assume that the vacuum $|G\rangle$ is easy to prepare which requires sufficiently large $\omega_{\rm res}$. Population transfer $|G\rangle \to |U\rangle$ is then performed via $|E\rangle$. As the "Stokes field" $A_u(t)$ is replaced by the vacuum field of the cavity a single phonon is emitted into the resonator during STIRAP.

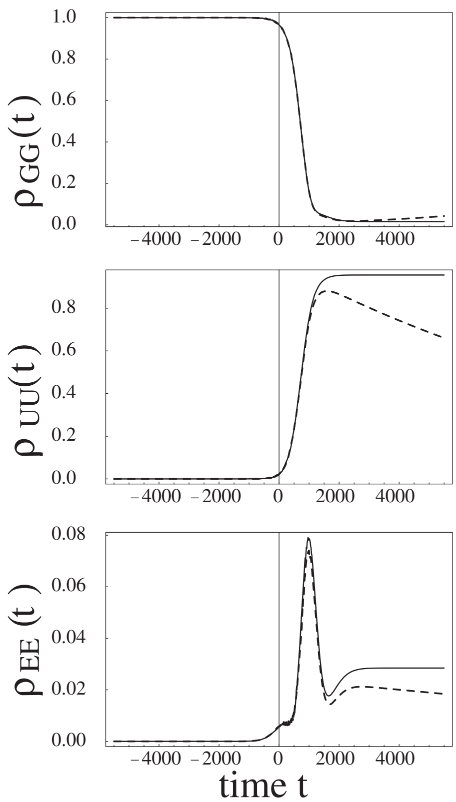

FIG. 4. Level population for the CPB-resonator setup without decoherence (solid lines) and in the presence of decoherence (dashed lines). Parameters are $n_{g0}(t)+n_x(t)=0.03$, $\max[A_g(t),\lambda(t)] =0.05$, $\gamma_u=4.4\times10^{-5}$, $\tilde{\gamma}=2.6\times10^{-4}$, and $Q=\omega_{\rm res}/\kappa=5.0\times10^3$.

Preparation of $|G\rangle$ requires $\omega_{\rm res} > k_B T/\hbar$ corresponding to $\omega_{\rm res} > 2\pi\times 1$ GHz for $T \lesssim 30$ mK, which is at the limit of present-day technology.[34] The oscillator needs to be resonant with the CPB transition $u \to e$. Using $E_C \approx 2E_J \sim 35$ μeV and $n_{g0} \sim 0.03$ it is possible to have $\omega_{\rm res} \sim 2\pi\times 1.5$ GHz. Near the "sweet spot" $n_{g0}=0$ one may hope to achieve similar decoherence effects as in the experiments in Refs. 2 and 25 and, at the same time, to generate the appropriate level spacings. Taking into account a finite quality factor of the resonator $Q=\omega_{\rm res}/\kappa=5.0\times10^3$ we can numerically evaluate the time evolution of the coupled system. Note that now it is necessary to include the state $|u,0\rangle$ which is not part of the STIRAP scheme [see Fig. 3(b)] but contributes to reduce coherence of the population transfer. It can be seen that a highly efficient transfer of the system to the state $|U\rangle$ should be feasible (cf. Fig. 4).

As to the detection, the direct measurement of the oscillator state would be desirable. However, it may be easier to probe the state $|U\rangle$ via a measurement of the CPB eigenstate. Either one probes the final state $|u\rangle$ or, viewing the system as a realization of the Jaynes-Cummings model,[26] one may detect Rabi oscillations between the states $|U\rangle$ and $|E\rangle$ induced by the cavity field. To this end, $\lambda(t)$ should be set to an appropriate value while $A_g=0$. Note that this detection requires high-quality resonators, and discrimination is necessary between the quantronium eigenstates $|u\rangle$ and $|e\rangle$.

Highly populated phonon states may be generated[22] by rotating the final state $|U\rangle=|u,1\rangle \to |g,1\rangle$, via a π pulse in the CPB while $\lambda(t)=0$. This initializes the system for another STIRAP transfer $|g,1\rangle \rightarrow (|e,1\rangle) \rightarrow |u,2\rangle$, etc.

V. CONCLUSIONS

We have analyzed methods of quantum control for the CPB in the charge-phase regime in presence of decoherence: the STIRAP protocol for the CPB (as an analog of a three-level atom) and a STIRAP-type protocol for Fock state generation in a nanomechanical oscillator coupled to the CPB circuit. The effect of the environment has been studied by means of a master equation. We have justified this phenomenological approach also for low-frequency noise that is characteristic to solid-state quantum-coherent systems.

In complete correspondence with previous studies we find that the STIRAP method is feasible in superconducting nanocircuits and that it turns out to be rather robust even in the presence of considerable amounts of noise. Our studies strongly support the statement that, even though high decay rates have to be expected for the third level in an artificial atom, there are protocols that work despite this apparent defect and that it may be worthwhile taking higher levels in consideration for practical applications. The STIRAP protocol with its numerous variations should be an excellent candidate toward this goal.

We conclude this discussion by emphasizing two of the main advantages of the STIRAP method. One of its important characteristics is that its efficiency depends only weakly on the strength of the couplings during the procedure and on details of the timing. This makes the procedure robust against fluctuations in particular in a solid-state environment. A further advantage is the flexibility of the method—there are many ways to adapt it to a given physical system. For example, the STIRAP protocol for the CPB can also be implemented by changing the driving frequencies[19] instead of changing the field amplitudes. Furthermore, the protocol to generate Fock states can even be realized with a *constant* CPB-resonator coupling[30]—e.g., for the fixed capacitive coupling between a CPB and an electrical resonator[32] or a coplanar waveguide resonator.[6,35] These few examples display that a wider consideration of STIRAP-type protocols in solid-state quantum-coherent systems may help to generate new solutions and applications also in this field of research.

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*Present address: Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany.

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